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422. Proposed by W. D. CAIRNS, Oberlin College.

Find a solution of the equation $x^{x\sqrt{x}} = (x\sqrt{x})^x$. [Adapted from *Godfrey & Siddon's Elementary Algebra.*]

Solved by Edward S. Ingham, F. L. Carmichael, H. T. Biglow, Elizabeth A. Davis. Walter C. Eells, D. H. Richert, H. C. Feemster, A. H. Holmes, Frank Irwin, George Y, Sosnow, Elbert H. Clarke, A. M. Harding, and the Proposer.

Solution by A. M. Harding, University of Arkansas.

If we take the logarithm of both members of the equation, we obtain

$$x\sqrt{x}\log x = x\log(x\sqrt{x}) = x\log(x)^{\frac{3}{2}} = \tfrac{3}{2}x\log x,$$

or

$$x(\sqrt{x} - \tfrac{3}{2})\log x = 0.$$

Hence $x = 0$, $\log x = 0$, and $\sqrt{x} - \frac{3}{2} = 0$. Therefore $x = 0$, $x = 1$, and $x = \frac{9}{4}$. If we substitute $x = 0$, both members of the given equation take the indeterminate form 0^0. In order to show that $x = 0$ is a root, we proceed as follows:

Since $x^{x^{3/2}} = (x^{3/2})^x = x^{\frac{3}{2}x}$, we have $x^{(x^{3/2}-\frac{3}{2}x)} = 1$. Hence we must show that $[x^{(x^{3/2}-\frac{3}{2}x)}] \doteq 1$ as $x \doteq 0$. Let $y = x^{(x^{3/2}-\frac{3}{2}x)}$, then $\log y = (x^{3/2} - \frac{3}{2}x)\log x$, or $\log y = \dfrac{\log x}{(x^{3/2} - \frac{3}{2}x)^{-1}} \doteq \dfrac{\infty}{\infty}$ as $x \doteq 0$. Differentiate numerator and denominator. Then

$$\lim_{x\doteq 0}\frac{\log x}{(x^{3/2} - \frac{3}{2}x)^{-1}} = \lim_{x\doteq 0}\frac{-2(x^{3/2} - \frac{3}{2}x)^2}{3(x^{3/2} - x)} = \lim_{x\doteq 0}\frac{-2x(x^{1/2} - \frac{3}{2})}{3(x^{1/2} - 1)} = 0.$$

Hence $\lim\limits_{x\doteq 0} y = \lim\limits_{x\doteq 0}[x^{(x^{3/2}-\frac{3}{2}x)}] = 1$, and $x = 0$ is a root of the given equation.

423. Proposed by ELBERT H. CLARKE, Purdue University.

Show that the following formula is true for all positive integral values of k. The parenthetical symbols are defined as being the binomial coefficients, and $\binom{k+1}{0} = 1$, by definition.

$$k^k\binom{k+1}{0} - (k-1)^k\binom{k+1}{1} + (k-2)^k\binom{k+1}{2} + \cdots$$
$$+ (-1)^{s-1}(k-s+1)^k\binom{k+1}{s-1} + \cdots + (-1)^{k-1}\binom{k+1}{k-1} = 1.$$

I. Solution by A. M. Harding, University of Arkansas.

There is a misprint in the last term of the left member of the equation as originally given. The exponent should be $k - 1$.

We have

$$e^{kx} = 1 + kx + \frac{k^2x^2}{2!} + \cdots + \frac{k^kx^k}{k!} + \cdots$$

$$e^{(k-1)x} = 1 + (k-1)x + \frac{(k-1)^2x^2}{2!} + \cdots + \frac{(k-1)^kx^k}{k!} + \cdots$$

$$\cdots = \cdots$$

$$e^{(k-s+1)x} = 1 + (k-s+1)x + \frac{(k-s+1)^2x^2}{2!} + \cdots + \frac{(k-s+1)^kx^k}{k!} + \cdots$$

$$\cdots = \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^k}{k!} + \cdots.$$

Multiply these equations in order by $\binom{k+1}{0}, -\binom{k+1}{1}, \cdots,$

$(-1)^{s-1}\binom{k+1}{s-1}, \cdots, (-1)^{k-1}\binom{k+1}{k-1}$ and add.

Denote the left member of the given equation by $F(k)$.

It will be seen that the coefficient of x^k in the above sum is $\frac{F(k)}{k!}$. Hence $\frac{F(k)}{k!}$ is the coefficient of x^k in the expansion

$$e^{kx} - \binom{k+1}{1} e^{(k-1)x} + \binom{k+1}{2} e^{(k-2)x} - \cdots + (-1)^{s-1}\binom{k+1}{s-1} e^{(k-s+1)x} + \cdots + (-1)^{k-1}\binom{k+1}{k-1} e^{x}.$$

This expansion can be written in the form

$$\begin{aligned} &e^{kx}\left[1 - \binom{k+1}{1} e^{-x} + \binom{k+1}{2} e^{-2x} - \cdots + (-1)^{s-1}\binom{k+1}{s-1} e^{(-s+1)x} + \cdots + (-1)^{k-1}\binom{k+1}{k-1} e^{-(k-1)x}\right] \\ &= e^{kx}\left[(1 - e^{-x})^{k+1} - (-1)^{k}\binom{k+1}{k} e^{-kx} - (-1)^{k+1}\binom{k+1}{k+1} e^{-(k+1)x}\right] \\ &= e^{kx}(1 - e^{-x})^{k+1} + (-1)^{k+1}(k + 1 - e^{-x}) \\ &= \left(1 + kx + \frac{k^2x^2}{2!} + \cdots\right)\left(x - \frac{x^2}{2!} + \frac{x^3}{3!} - \cdots\right)^{k+1} \\ &\qquad + (-1)^{k+1}\left(k + x - \frac{x^2}{2!} + \cdots + (-1)^{k+1}\frac{x^k}{k!} + \cdots\right). \end{aligned}$$

The coefficient of x^k in this expression is $\frac{(-1)^{2k+2}}{k!}$.

Hence $\frac{F(k)}{k!} = \frac{1}{k!}$. Hence $F(k) = 1$.

II. Solution by A. M. Kenyon, Purdue University.

This formula is a special case under the theorem:

If the coefficients of the binomial expansion of $(x - y)^n$, $n = 1, 2, 3, \cdots$, be multiplied, term by term, by the mth power, $m = 0, 1, 2, \cdots$, of the terms of any arithmetic progression, the sum of the products will vanish if $m < n$.

$$\sum_{t=0}^{n} (-1)^t \binom{n}{t} (a + td)^m = (-d)^m \sum_{t=0}^{n} (-1)^t \binom{n}{t} (x - t)^m,$$

where $x = -a/d$. The sum on the right is equal to

$$\sum_{s=0}^{m}(-1)^s\binom{m}{s}x^{m-s}\sum_{t=0}^{n}(-1)^t\binom{n}{t}t^s,$$

of which the inner sum vanishes for $s < n$, therefore for every s when $m < n$.

If now we set $k + 1$ for n, k for a, -1 for d, and k for m, we have

$$\sum_{t=0}^{k+1}(-1)^t\binom{k+1}{t}(k-1)^k = 0,$$

that is,

$$\binom{k+1}{0}k^k - \binom{k+1}{1}(k-1)^k + \cdots + (-1)^{k-1}\binom{k+1}{k-1}1^k + 0 - 1 = 0.$$

Also solved by S. A. Joffe.

GEOMETRY.

448. Proposed by S. W. REAVES, University of Oklahoma.

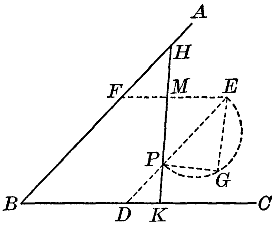
Through a given point P within a given angle to draw a line which shall form with the sides of the angle a triangle of a given area [Well's *New Plane Geometry* (1909), p. 153].

Solved in various ways by Horace Olson, Clifford N. Mills, George Y. Sosnow, Nathan Altshiller, and A. Holmes.

Solution by H. O. Hanson, East Elmhurst, N. Y.

Let $\angle ABC$ be the given angle, and P the given point within the angle.

Through P draw the line DE parallel to BA, and draw the line FE parallel to BC so that $BFED$ forms a parallelogram of the given area. On PE as hypothenuse construct the right triangle PEG so that $PG = PD$. Lay off $FH = GE$, and draw a line through H and P meeting BC at K and FE at M.



Then $\triangle BHK$ is the triangle required.

For, in the right triangle PEG, we have,

$$\overline{PG}^2 + \overline{GE}^2 = \overline{PE}^2,$$

or, since $PG = PD$, and $GE = FH$,

$$\overline{PD}^2 + \overline{FH}^2 = \overline{PE}^2.$$